Chapter 4
Analysis & Collimator

In this, and the following three chapters, each component of the spectrograph is analysed using a simple first order ray-trace method. A reasonable estimate of the fibre image on the detector can then be found including the effects of chromatic aberrations in the collimator and camera lenses. The optical through-put of each component is also considered.

The prism and grating are analysed in Chapter 5, the camera / detector system in Chapter 6 as well as the theoretical image of the fibre on the detector. The results of all this are combined in Chapter 7 to estimate the resolution under various operating conditions. The overall optical efficiency and exposure times are also calculated.

4.1 Analysis Methods and Assumptions

With the use of geometric optics and light “rays", a “system function" can be found for a given optical system (or component). In theory this should allow the form of the image to be determined for any object, however except for very simple aberration free systems, the amount of computation required is immense. Although our system contains only linear elements and the angular field sizes are small (compared to the design parameters of the lenses used), both the prism and the echelle grating are not axially symmetric. This precludes the use of standard Gaussian optics and the associated 2×2 matrix operators, because skew rays must also be considered. Fortunately the matrix optical techniques can be extended to include skew rays and it is this 4×4 matrix representation, based the work of Attard [1984], that will be used in this analysis.

The system matrix for each component can be found by determining the output ray from the component for an arbitrary input ray. Each ray is defined in terms of its position and orientation with respect to a reference or principal ray (which corresponds to the optic axis in simple lens systems). This is the ray that passes from the centre of the fibre along the optic axis of the collimator. Assuming that the spectrograph is correctly aligned, the wavelength for this ray can be chosen so that:

(a) it enters the centre of the prism face and passes through at minimum deviation, and

(b) it corresponds to the central wavelength of some order (which one is not important), having been reflected from the middle of the grating.

The ray will (well should anyway) pass along the optic axis of the camera and hence onto the centre of the detector. At other wavelengths the fibre will be imaged away from the centre of the detector but because our system is linear and the angular field sizes are small, the form of the image should not vary by much, i.e. the system is *isoplanic*.

A cartesian-altazimuth coordinate system was chosen so that one of the axes corresponds to the principal ray. This is the z axis. The x axis corresponds with the cross-dispersion direction (horizontal) and the y axis with the direction of dispersion (vertical), being more positive as λ increases. Rays are defined as 4×1 column matrices (vectors) in terms of their position, x,y , altitude angle, ϕ_x, and azimuth angle, ϕ_y. The extended 4×4 matrix formulation for the Gaussian ray trace is then:

$$\begin{vmatrix} x' \\ y' \\ \phi'_x \\ \phi'_y \end{vmatrix} = \begin{vmatrix} A_1 & A_2 & A_3 & A_4 \\ B_1 & B_2 & B_3 & B_4 \\ C_1 & C_2 & C_3 & C_4 \\ D_1 & D_2 & D_3 & D_4 \end{vmatrix} \begin{vmatrix} x \\ y \\ \phi_x \\ \phi_y \end{vmatrix} \tag{4.1}$$

For example the translation matrix T for a translation of length t is:

$$T = \begin{vmatrix} 1 & 0 & t & 0 \\ 0 & 1 & 0 & t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \tag{4.2}$$

and the refraction matrix R for a thin lens of focal effective focal length f is:

$$R = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1/f & 0 & 1 & 0 \\ 0 & -1/f & 0 & 1 \end{vmatrix} \tag{4.3}$$

Both of these matrices will be used later for the collimator and camera lenses.

The system matrix for the spectrograph, S, can be calculated once the matrix for each component is found:

$$S = KT_3GT_2PT_1C \tag{4.4}$$

Where the component matrices are: K: camera, G: grating, P: prism, and C: collimator. The translation matrices T correspond to the component separations. If we define light "rays" at the object plane (fibre end) and image plane (detector) as $\vec{o}$ and $\vec{d}$ respectively, then:

$$\vec{d} = S\vec{o} \tag{4.5}$$

Before starting the analysis on the collimator a couple of important assumptions used in the analysis must be presented.

1. The image of a point source will change only in position, **not** in functional form (isoplanic), and the system can be considered as aberration free if the chromatic aberrations are considered as small focusing errors. This is important because it allows the use of the analysis results at any wavelength (i.e. at any angle of dispersion) once variations in the "focusing errors" are included. The size and shape if the fibre image effects both the resolving power of the spectrograph and exposure times.

2. The angle, ϕ, that any ray makes with the principal ray (z axis) is small and if the distances between optical components, d, is also sufficiently small then $d \times \phi \approx 0$. This means that the translation matrices, T, in equation 4.4 can be approximated by unit matrices and hence ignored. This will be justified in Chapter 6.

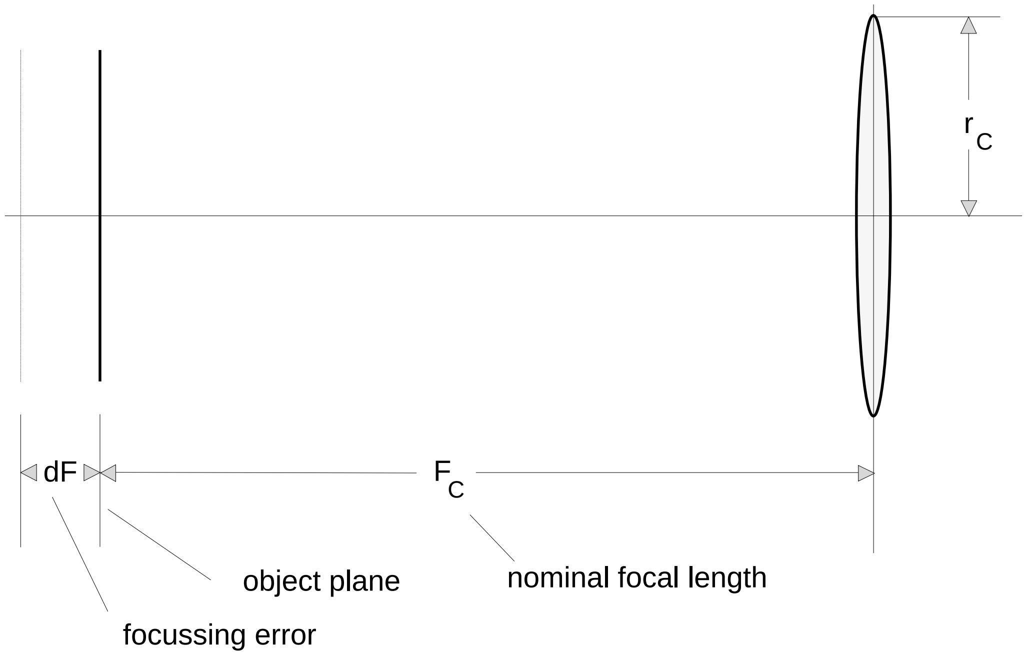

Figure 4.1: A simple collimating lens with focusing error.

4.2 The Collimator Matrix C

The focal length of a simple achromat is typically 0.2% greater at wavelengths near 500nm than at the ends of the visible spectrum at 400 or 800nm. Tests on a number of 400mm f2.8 commercial objective lenses by Baudrand and Böhm [1992], however, yielded values for this focussing error at less than 0.02%. In this spectrograph we will be using a relatively inexpensive Kimunor 400mm f6.3 objective lens. Equipment for the direct measurement of the size of the relative error, was unfortunately unavailable, so considering the above mentioned results a value of approximately 0.03% will be assumed. Note that this value will also be used, in Chapter 6, for the camera lens.

Figure 4.1 illustrates the geometry of a simple collimating lens including a small focussing error.

Using the terminology defined in Figure 4.1, the path of a light ray from the object plane (optical fibre end) to the output of the collimator can be described by a translation followed by refraction. The collimator matrix, C, can therefore be derived from:

$$C = R_{(F_c+dF)}\, T_{F_c} \tag{4.6}$$

If we let

$$\varepsilon_c = \frac{dF}{F_c + dF} \tag{4.7}$$

then:

$$C \approx \begin{vmatrix} 1 & 0 & F_c & 0 \\ 0 & 1 & 0 & F_c \\ \frac{-1}{F_c(1+\varepsilon_c)} & 0 & \varepsilon_c & 0 \\ 0 & \frac{-1}{F_c(1+\varepsilon_c)} & 0 & \varepsilon_c \end{vmatrix} \tag{4.8}$$

4.3 Collimator Through-put

The relative transmission of the collimator lens was measured over a range of 400 to 800nm in 10nm steps. Next a laser diode and photo-voltaic cell were employed to measure the absolute transmission at 670nm with a the result of 85±3%. This result was used to scale the relative values to obtain an estimate for the transmission over the visible spectrum and is shown in Figure 4.2. Full details of the testing procedures are described in Appendix B.

The “scattering" observed in the graph is mainly noise in the measurements taken. We are only concerned, however, with the general trend of the data and in this case the transmission for the collimator can be adequately represented by a single linear segment:

$$\text{For } \lambda \text{ in } \mu\text{m} \qquad \tau_{coll} \approx 1.05 - 0.325\lambda \tag{4.9}$$

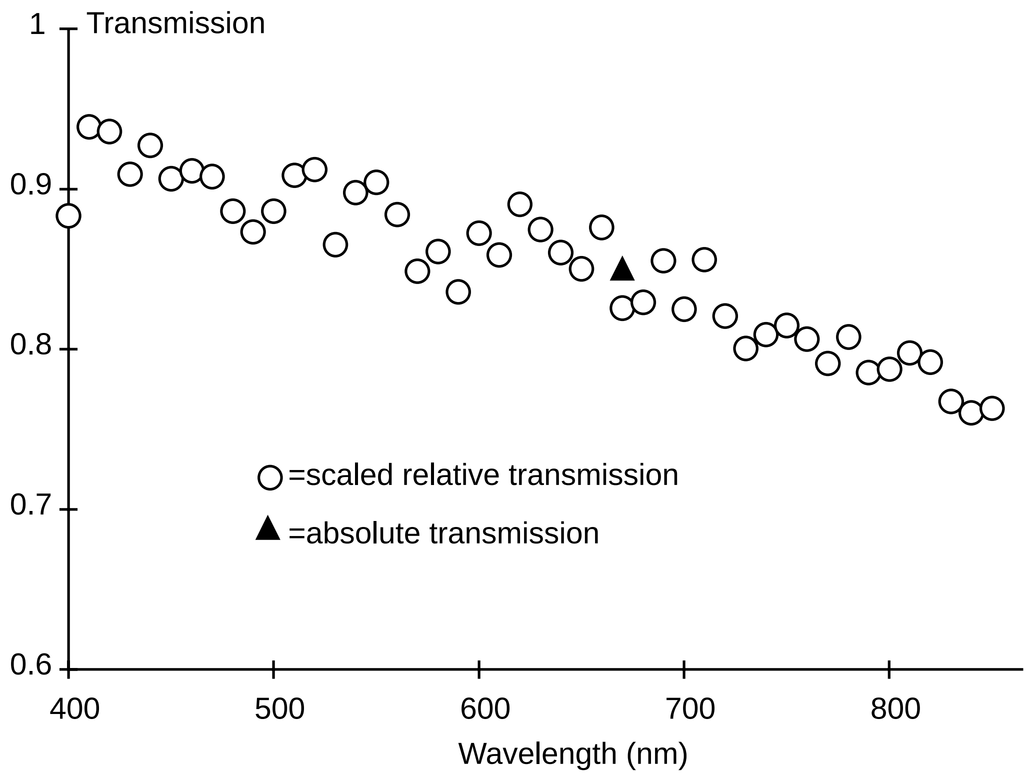

Figure 4.2: Transmission for 400mm f6.3 Kimunor lens.